```
disp(datetime)
```

```
05-Dec-2020 10:28:01
```

Buckling Venus Lithosphere

This script documents the models and results reported in the manuscript entitled "Felsic Tesserae on Venus Permitted by Lithospheric Deformation Models," by Resor et al., submitted to JGR Planets. The script accesses mineral creep data and rock compositions in the Excel file *Creep_database.xlsx.* These data are used to generate viscous buckling plots and lithosphereic strength envelopes (AKA Christmas tree diagrams) for single mineral lithospheres and polymineralic lithospheres.

Monomineralic lithosphere

Read in creep parameters

Estimates of dry creep parameters are avaiable for olivine (Hirth and Kolstead, 2003), diopside (Dimanov and Dressen, 2005), and anorthite (Rybacki et al., 2006). The dry quartz flow law is modified from Hirth et al. (2001) using the results of dry experiments of Barberry (2017).

```
% read mineral creep parameters from file
% columns: dry olivine, dry diopside, dry anorthite, quartz
% rows: A* n Q V (m^3/mol) rho
params = xlsread('./Resoretal_minrx_param.xlsx',...
    'minerals','B2:H5');

% seperate into individal parameters
Astar = params(:,1);
n = params(:,2);
Q = params(:,3);
V = params(:,4);
rho = params(:,5);
fugExp = params(:,6);
cOHExp = params(:,7);
name = {'olivine', 'diopside', 'anorthite', 'quartz'};

% assumed pressure for laws with activation volume effect
P = 300e6; % Pa

% assumed water concentration and fugacity
fug = 200; % MPa
cOH = 10; % ppm H/Si

% adjust Q for activation volume effect, if included
Q = Q + P*V;

% adjust Astar for water concentration and fugacity terms (quartz only)
% using fugacity term of Hirth et al. 2001 and cOH term of Barbery 2017
Astar = Astar .* fug.^fugExp .* cOH.^cOHExp;

% adjust Astar values for triaxial deformation (see Ranalli, 95, p. 76
% or Burov 2015, eq. 18)
Astar = Astar .* 3.^((n+1)./2) ./2;
```

Define additional relevant constants

```
% gravity and surface temperature of Venus
g=8.87; % m/s^2
Ts=740; % deg K

R=8.31446; % J/K/mol (NIST 2014)

% define tectonic regime for strength envelope calculation
phi='r'; % reverse-faulting

% define dominant fold wavelength and uncertainty bounds based on new
% values cited in manuscript text
lam = [14.4 10.1 18.7];

% Time between cratering events in seconds
Ct = 12e6*365.24219*86400;

% ln of fold amplification assuming initial topography of 10 m and
% amplitude of 500 m
lnAmp = log(500/10);
```

Calculate buckling instability

```
% plot buckling instabilities
% set up ranges of strain rate and thermal gradient to iterate over
% range of likely values (see explanation in manuscript text)
logexx = -17:0.05:-14;
thKkm = 5:0.5:80;

% time bounds 1% strain
logexxtime = log10(0.01/(365.24219*86400) * 1./(10.^[-14 -17]));

% create variable to store numerical results
MinLvalsPD = zeros(length(Astar), 2);
MinQvalsPD = zeros(length(Astar), 2);
MinOptvals = zeros(length(Astar), 3);
MinTHvals = zeros(length(Astar), 4);
MinQvals = zeros(length(Astar), 4);

% create blank figure
fig2 = figure(2);
fig2.Units = 'centimeters';
fig2.Position = [0, 0, 9.5, 23];
fig2.PaperUnits = 'centimeters';
fig2.PaperPosition = [0, 0, 9.5, 23];

for j = 1: length(Astar)
    % create empty matrices for solutions
    qvals=zeros(length(logexx),length(thKkm));
    Lvals=zeros(length(logexx),length(thKkm));
    Hvals=zeros(size(Lvals));
```

```
    % iterate over range of strain rates
    for u=1:length(logexx)
        Exx=-10^logexx(u);
        % iterate over range of thermal gradients
        for v=1:length(thKkm)
            th=thKkm(v)/1000; % convert to deg/m
            % Calculate buckling instability
            [qvals(u,v), Lvals(u,v), Hvals(u,v)] = perturb2(th, Ts, g, rho(j), Astar(j)
        end
    end

    subplot(7, 3, [3*j-1 3*j])
    [TH, E]=meshgrid(thKkm, logexx);

    % add mean and 1-sigma range as line and colored swath
    [~,~]=contourf(TH, E, Lvals/1000, lam(2:3));
    hold on
    Opt = contour(TH, E, Lvals/1000, [lam(1) lam(1)], 'k', 'LineWidth',2);
    QOpt = griddata(TH, E, qvals, Opt(1,2:end), Opt(2,2:end));
    EQ = log10(lnAmp./((QOpt+1)*Ct));
    k = find(EQ<=Opt(2,2:end),1);


    % plot color contours for all lvals
    [c,h]=contour(TH, E, Lvals/1000, (0:5:100));
    clabel(c,h, [5, 10, 15, 25, 40])

    % add contours of qvals
    [c,h]=contour(TH, E, imgaussfilt(qvals,3), [ 3 4 10:10:100], '--');
    clabel(c,h, [2:2:8 10:20:90])
    xlabel('thermal gradient (K/km)')
    ylabel('log strain rate (s^-^1)')
    title(name(j))

    % add points for envelope values
%     plot([thKkm1(1:2) thKkm1(j+2)], [logexx1(1:2) logexx1(j+2)], 'r.')

    % store results for present-day conditions
    MinLvalsPD(j,:) = Lvals(E==-17&TH==10)/1000;
    MinQvalsPD(j,:) = qvals(E==-17&TH==10);
    MinOptvals(j,:) = [Opt(:,k+1)' QOpt(k)];

    % store thermal gradient for dominant and upper/lower bound wavelengths
    % as well as 20 km wavelength (B&G) for log strain rates 10^-17 and 10^-15

    F = griddedInterpolant(TH', E', Lvals'/1000);
    Es = -15;
    options = optimset('Display','off'); % hide output
    for m = 1:length(Es)
        q = zeros(4,1);

        % find th and q for dominant wavelength
        fun = @(th)F(th,Es(m))-lam(1);
```

```
        med = fzero(fun,40,options);
        if isfinite(med); q(1) = qvals(E==Es(m)&TH==round(med)); end

        % find th and q for first quartile wavelength
        fun = @(th)F(th,Es(m))-lam(3);
        iqr1 = fzero(fun,40,options);
        if isfinite(iqr1); q(2) = qvals(E==Es(m)&TH==round(iqr1)); end

        % find th and q for third quartile wavelength
        fun = @(th)F(th,Es(m))-lam(2);
        iqr2 = fzero(fun,40,options);
        if isnan(iqr2)||iqr2>80; q(3) = qvals(E==Es(m)&TH==80);
            else q(3) = qvals(E==Es(m)&TH==round(iqr2));
        end

        % find th and q for B&G wavelength
        fun = @(th)F(th,Es(m))-20;
        bg = fzero(fun,40,options);
        if isfinite(bg); q(4) = qvals(E==Es(m)&TH==round(bg)); end

        MinTHvals(m+(j-1)*length(Es),:) = [med iqr1 iqr2 bg];
        MinQvals(m+(j-1)*length(Es),:) = q;
    end
    % plot points on graphs for present-day and best-fit values
    plot(10,-17, 'b.')
    plot(med, Es, 'r.')
    plot(Opt(1,k+1), Opt(2,k+1), 'k*')

    % set x axis labels to every 10 K/km
    ticks = 10:10:80;
    labels = cell(1,length(ticks));
    labels(1:2:length(ticks)) = num2cell(ticks(1:2:length(ticks)));
    xticks(ticks)
    xticklabels(labels)
    ax = gca;
    pos = ax.Position;
    ax.Position = [pos(1)+0.2*pos(3), pos(2), 0.8*pos(3), pos(4)];

    % add second y axis with strain times on log scale
    ax2 = axes('Position', ax.Position,...
    'YLim', logexxtime,...
    'Ydir', 'reverse',...
    'YTick', [5 6 7],...
    'YTicklabel', [5 6 7],...
    'XTick',[],...
    'YAxisLocation','right',...
    'XaxisLocation','top',...
    'Color','none');

end
```

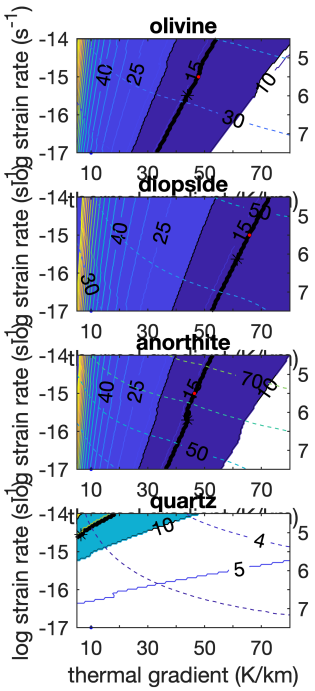


Figure 2. Contour plots of dominant wavelength (solid contours) and growth factor (dashed contours) for single layer buckling instability models. Plots are for monomineralic a) olivine, b) diopside, c) anorthite, and d) quartz lithospheres.

```
%save as vector pdf
% fig2.Renderer='Painters';
% saveas(fig2,'fig2','pdf')
```

```
% display tables of results

table(MinTHvals,MinQvals,'RowNames', name, 'VariableNames', {'ThermalGradients', 'Growt
```

```
ans = 4×2 table
```

...

	ThermalGradients			
1 olivine	47.8482	35.3599	70.4345	33.0496
2 diopside	65.8216	48.3334	NaN	45.0230
3 anorthite	46.4400	35.1096	70.7703	32.1957
4 quartz	NaN	NaN	10.8616	NaN

Table A. Geotherms (K/km) and Growth factors for single phase models with log strain rates of -15 for dominant wavelength (coumn 1), interquartile range (columns 2, 3) and 20 km wavelength (column 4).

```
table(MinOptvals,'RowNames', name)
```

```
ans = 4×1 table
```

	MinOptvals		
1 olivine	44.0000	-15.4848	30.9308
2 diopside	61.5000	-15.6322	44.1006
3 anorthite	44.0129	-15.7000	56.2030
4 quartz	6.4597	-14.5500	2.8192

Plot Strength Envelopes

Strength envelopes for end-member parameters are plotted to illustrate the effects of strain rate and geotherm.

```
% create christmas tree plot for representative values
% set up ranges of strain rate and thermal gradient to iterate over
% values based on present conditions (1) and this study. Values for
% this study are automatically passed on from section above, except
% for quartz, which is set to 5 K/km
logexx1 = [-17, -15, -15,  -15,  -15, MinOptvals(:,2)'];
thKkm1 = [10, MinTHvals(1:3,1)', 5, MinOptvals(:,1)'];

% Create empty array to store numerical results
MinResults = zeros(length(Astar)*3, 6);

% loop over range of materials
for j = 1: length(Astar)
    subplot(7, 3, (3*j-2))
    hold on
    th = [thKkm1(1) thKkm1(j+1) thKkm1(j+length(Astar)+1)]./1000;
    Exx = 10.^[logexx1(1) logexx1(j+1) logexx1(j+length(Astar)+1)];
    for k = 1:length(th)
        [H, gam, eta, z, env] = bdt_plot(th(k), Ts, g, rho(j), Astar(j), n(j), Q(j), ab
        plot(-env/1000, -z/1000)
        dx = (z(2)-z(1));
        IntStrength = sum(dx.*(env(1:end-1) + env(2:end))./2) * 1e6; %N/m
```

```
        MinResults((j-1)*length(th)+k,:) = [log10(Exx(k)), th(k)*1000, H/1000, env(z==H
    end
    hold off
    xlabel('diff. stress (comp. +)')
    ylabel('depth (km)')
%     legend([num2str(log10(Exx(1))),', ',num2str(th(1)*1000)],...
%         [num2str(log10(Exx(2))),', ',num2str(th(2)*1000)],...
%         [num2str(log10(Exx(3))),', ',num2str(th(3)*1000)],...
%         'Location', 'southeast')
    title(name(j));
    axis([0 1.2 -60 0])
    yticks([-60, -30, 0])
    xticks([0, 0.5, 1])
end
```

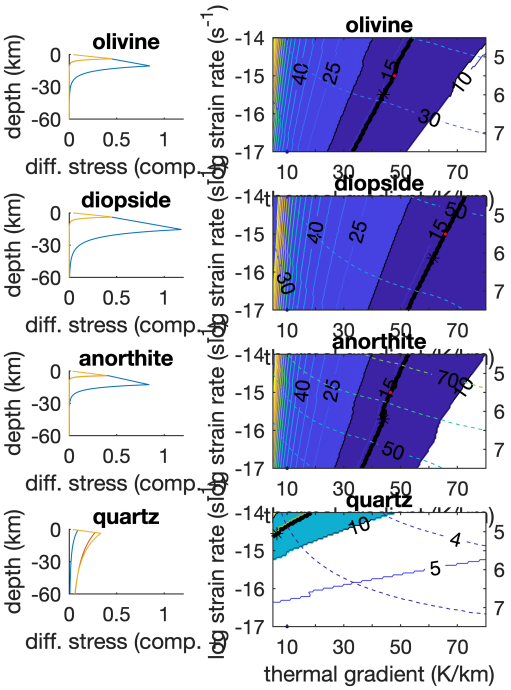


Figure 2. Strength envelopes for monomineralic lithospheres. Plots are for monomineralic a) olivine, b) diopside, c)anorthite, and d) quartz lithospheres. Red dots are preferred model. Blue dots are present-day conditions.

```
table(MinResults, 'RowNames',...
    [name(1) 'a' '1' name(2) 'b' '2' name(3) 'c' '3' name(4) 'd' '4'])
```

```
ans = 12×1 table
```

	MinResults					
1 olivine	-17.0000	10.0000	10.5803	-848.5697	4.2428e+25	0.0003
2 a	-15.0000	47.8482	3.9803	-437.1953	2.1860e+23	0.0010
3 1	-15.4848	44.0000	3.9803	-442.4752	6.7558e+23	0.0010
4 diopside	-17.0000	10.0000	15.4750	-1.1775e+03	5.8874e+25	0.0002
5 b	-15.0000	65.8216	3.9750	-446.2536	2.2313e+23	0.0010
6 2	-15.6322	61.5000	3.9750	-445.6199	9.5521e+23	0.0010
7 anorthite	-17.0000	10.0000	12.6279	-813.4685	4.0673e+25	0.0003
8 c	-15.0000	46.4400	4.2279	-400.0882	2.0004e+23	0.0014
9 3	-15.7000	44.0129	4.1279	-363.5557	9.1105e+23	0.0013
10 quartz	-17.0000	10.0000	0.7443	-90.7517	4.5376e+24	0.0001
11 d	-15.0000	5.0000	2.8443	-273.3110	1.3666e+23	0.0000
12 4	-14.5500	6.4597	3.5443	-333.1067	5.9095e+22	0.0000

Table B. log strain rate (1/sec), geotherm (K/km), BDT (km), Stress at

BDT (MPa), viscosity at BDT (PaSec) and e-folding depth for various single phase models

```
% pos = fig2.Position;
% axesj.Units = 'centimeters';
% axesj.OuterPosition = [0, -(j-1)*2.875, 3, 2.75];
```

Plot viscosity vs depth for power law vs exponential approximation

```
figS1 = figure(4);

% loop over range of materials
for j = 1: length(Astar)
    subplot(2, 2, (j))
    hold on
    z = MinResults(j*3-1,3)*1000:100:10000;
    T = Ts + z .* MinResults(j*3-1,2)./1000;
    visc = 1e6 .* 1/2*Astar(j).^(-1/n(j))...
        .* abs(10.^(MinResults(j*3-1,1))).^-(1-1/n(j))...
```

```
        .* exp(Q(j)./(n(j).*R.*T));
%     eta=1e6*1/2*Astar^(-1/n)*abs(Exx)^-(1-1/n)*exp(Q/(n*R*(Ts+th*H)))
    plot((visc),-z/1000,'r-')
    viscHat = MinResults(j*3-1,5).*exp(-MinResults(j*3-1,6).*(z-MinResults(j*3-1,3).*10
    hold on
    plot((viscHat),-z/1000,'r:')
end
```

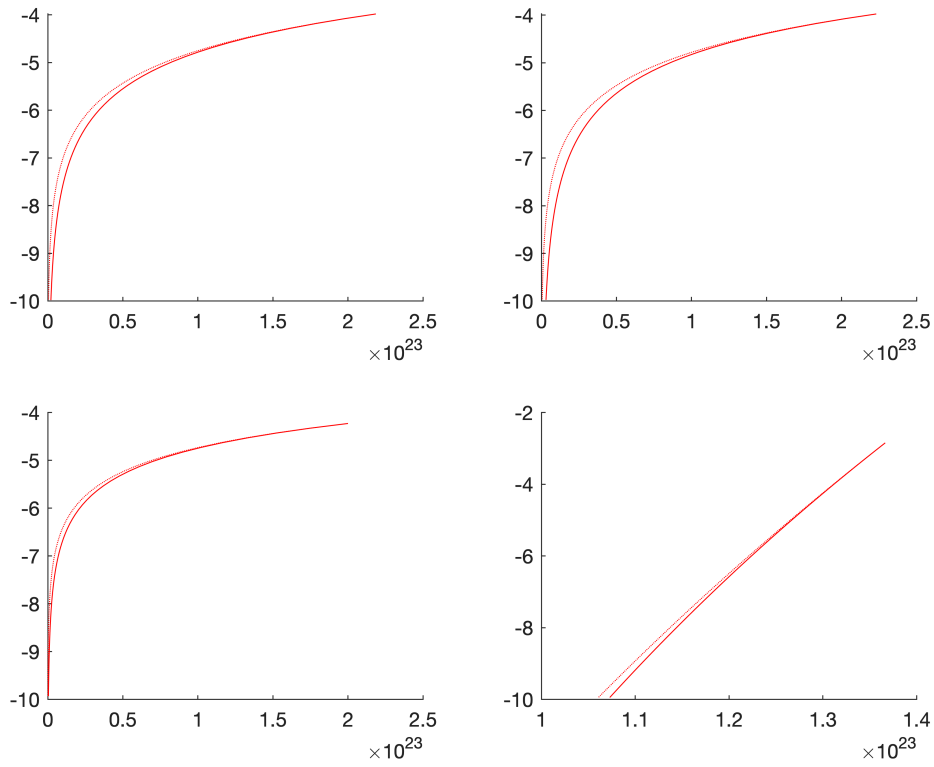


Figure S1. Viscosity decay with depth for single phase models. Solid line is based on flow law (eq. 2) and dashed line is the approximation used in the model (eq. 3)

Polymineralic lithosphere

Dry whole rock data are only available for Basaltic compositions (Columbia and Maryland diabase, Mackwell et al., 1998). We simulate whole rock deformatin of polymineralic rocks using the Minimized Power Geometric model of Huet et al., 2014).

We model a suite of mafic to felsic compositions from the Laramie Anorthosite Complex (Anderson, 1995; Anderson et al., 2003). Dry mineral data are limited. We group minerals based on observation of their relative strengths when data are not available.

```
% create empty matrix for rock mineralogy and then load in values from
% excel spreadsheet

% define number of synthetic rocks to be analyzed
```

```
NumRocks = 4;

% create empty vector for modal mineral values
rocks = zeros(14,NumRocks);
name2 = {'Columbia Diabase (exp)', 'Columbia Diabase (synth)', 'Venera 14',...
    'quartz monzonite', 'granite'};
rocks(:,1) = xlsread('./Resoretal_minrx_param.xlsx',...
    'compositions','AU3:AU16'); % Columbia diabase, Mackwell et al., 1998
rocks(:,2) = xlsread('./Resoretal_minrx_param.xlsx',...
    'compositions','AW3:AW16'); % Venera 14 CIPW,
rocks(:,3) = xlsread('./Resoretal_minrx_param.xlsx',...
    'compositions','AO3:AO16'); % Quartz Monzonite MPK81, Anderson et al., 1995, 2003
rocks(:,4) = xlsread('./Resoretal_minrx_param.xlsx',...
    'compositions','AS3:AS16'); % Granite MPK78, Anderson et al., 1995, 2003

% combine similar minerals
simple = zeros(size(params,1),size(rocks,2));
simple(3,:) = rocks(1,:) + rocks(2,:); % combine all feldspars
simple(1,:) = rocks(3,:); % olivine
simple(2,:) = rocks(4,:) + rocks(5,:) + rocks(6,:)+ rocks(9,:); % pyroxene and hornbler
simple(4,:) = rocks(7,:); % quartz

% renormalize
simple = simple./repmat(sum(simple,1),4,1);

% calculate apparent (model) density
rhobar = zeros(size(simple,2)+1,1); % add one for expreimental sample
rhobar(2:end) = sum(simple.*repmat(rho,1,NumRocks),1);

% Calculate whole rock creep parameters from mineral data using MPGe mixing

% set up empty vectors for creep paramaters
nbar = zeros(size(simple,2)+1,1); % add one for expreimental sample
Qbar = nbar;
Abar = nbar;

% loop over rocks and calculate average values
for j = 2: size(simple,2)+1 % add one for expreimental sample
    [nbar(j), Qbar(j), Abar(j)] = MPGe(simple(:,j-1), n, Q, Astar);
end

% add in experimental values for Columbia diabase
Columbia = xlsread('./Resoretal_minrx_param.xlsx',...
    'rocks','B2:D2');
nbar(1) = Columbia(2);
Qbar(1) = Columbia(3);
Abar(1) = Columbia(1);
Abar(1) = Abar(1) .* 3.^((nbar(1)+1)./2) ./2;
rhobar(1) = 3000;
```

And calculate buckling instability

```
% plot buckling instabilities
% set up ranges of strain rate and thermal gradient to iterate over
% values used in 2010 LPSC poster
logexx = -17:0.05:-14;
thKkm = 5:0.5:80;

% create variable to store numerical results
RockLvalsPD = zeros(length(Astar), 2);
RockQvalsPD = zeros(length(Astar), 2);
RockOptvals = zeros(length(Astar), 3);
RockTHvals = zeros(length(Astar), 4);
RockQvals = zeros(length(Astar), 4);

% create blank figure
fig3 = figure(3);
fig3.Units = 'centimeters';
fig3.Position = [0, 0, 9.5, 23];
fig3.PaperUnits = 'centimeters';
fig3.PaperPosition = [0, 0, 9.5, 23];

for j = 1: length(Abar)
    % create empty matrices for solutions
    qvals=zeros(length(logexx),length(thKkm));
    Lvals=zeros(length(logexx),length(thKkm));
    Hvals=zeros(size(Lvals));

    % iterate over range of strain rates
    for u=1:length(logexx)
        Exx=-10^logexx(u);
        % iterate over range of thermal gradients
        for v=1:length(thKkm)
            th=thKkm(v)/1000; % convert to deg/m
            % Calculate buckling instability
            [qvals(u,v), Lvals(u,v), Hvals(u,v)] = perturb2(th, Ts, g, rhobar(j), Abar
        end
    end

    subplot(7, 3, [3*j-1 3*j])
    [TH, E]=meshgrid(thKkm, logexx);

    % add mean and 1-sigma range as line and colored swath
    [~,~]=contourf(TH, E, Lvals/1000, lam(2:3));
    hold on
    Opt = contour(TH, E, Lvals/1000, [lam(1) lam(1)], 'k', 'LineWidth',2);
    QOpt = griddata(TH, E, qvals, Opt(1,2:end), Opt(2,2:end));
    EQ = log10(lnAmp./((QOpt+1)*Ct));
    k = find(EQ<=Opt(2,2:end),1);

    % plot color contours for all lvals
    [c,h]=contour(TH, E, Lvals/1000, (0:5:100));
    clabel(c,h, [5, 10, 15, 25, 40])

    % add contours of qvals
```

```
    [c,h]=contour(TH, E, imgaussfilt(qvals,3), (0:10:100), '--');
    clabel(c,h, [2:2:8 10:20:90])
    xlabel('thermal gradient (K/km)')
    ylabel('log strain rate (s^-^1)')
    title(name2(j))

%     % add points for envelope values
%     plot([thKkm2(1:2) thKkm2(j+2)], [logexx2(1:2) logexx2(j+2)], 'r.')

    % store results for present-day and B&G Tessera
    RockLvalsPD(j,:) = Lvals(E==-17&TH==10)/1000;
    RockQvalsPD(j,:) = qvals(E==-17&TH==10);
    RockOptvals(j,:) = [Opt(:,k+1)' QOpt(k)];

    % store median and IQR values for specific strain rates
    F = griddedInterpolant(TH', E', Lvals'/1000);
    Es = -15; % for range of values: min(logexx):1:max(logexx);
    for m = 1:length(Es)
        q=zeros(4,1);

        % find th and q for dominant wavelength
        fun = @(th)F(th,Es(m))-lam(1);
        med = fzero(fun,40,options);
        if isfinite(med); q(1) = qvals(E==Es(m)&TH==round(med)); end

        % find th and q for first quartile wavelength
        fun = @(th)F(th,Es(m))-lam(3);
        iqr1 = fzero(fun,40,options);
        if isfinite(iqr1); q(2) = qvals(E==Es(m)&TH==round(iqr1)); end

        % find th and q for third quartile wavelength
        fun = @(th)F(th,Es(m))-lam(2);
        iqr2 = fzero(fun,40,options);
        if isnan(iqr2)||iqr2>80; q(3) = qvals(E==Es(m)&TH==80);
            else q(3) = qvals(E==Es(m)&TH==round(iqr2));
        end

        % find th and q for B&G (20 km) wavelength
        fun = @(th)F(th,Es(m))-20;
        bg = fzero(fun,40,options);
        if isfinite(bg); q(4) = qvals(E==Es(m)&TH==round(bg)); end

        RockTHvals(m+(j-1)*length(Es),:) = [med iqr1 iqr2 bg];
        RockQvals(m+(j-1)*length(Es),:) = q;
    end

    % plot points on graphs for present-day and best-fit values
    plot(10,-17, 'b.')
    plot(med, Es, 'r.')
    plot(Opt(1,k+1), Opt(2,k+1), 'k*')

        % set x axis labels to every 10 K/km
    ticks = 10:10:80;
    labels = cell(1,length(ticks));
```

```
    labels(1:2:length(ticks)) = num2cell(ticks(1:2:length(ticks)));
    xticks(ticks)
    xticklabels(labels)
    ax = gca;
    pos = ax.Position;
    ax.Position = [pos(1)+0.2*pos(3), pos(2), 0.8*pos(3), pos(4)];

    % add second y axis with strain times on log scale
    ax2 = axes('Position', ax.Position,...
    'YLim', logexxtime,...
    'Ydir', 'reverse',...
    'YTick', [5 6 7],...
    'YTicklabel', [5 6 7],...
    'XTick',[],...
    'YAxisLocation','right',...
    'XaxisLocation','top',...
    'Color','none');
end
%
% pos = fig.Position;
% fig.Position = [pos(1:2), 400, pos(3)];
%
fig3.Renderer='Painters';
saveas(fig3,'fig3','pdf')
```

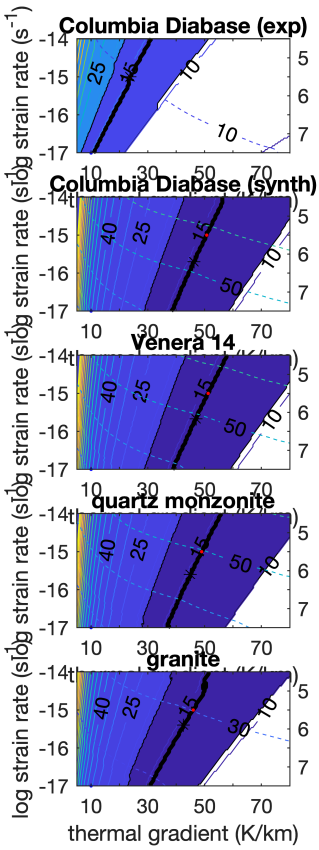


Figure 3. Contour plots of dominant wavelength (solid contours) and growth factor (dashed contours) for single layer buckling instability models. Plots are for polymineralic a) diabase (natrual), b) diabase (model), c) Venera 14 CIPW, and d) quartz monzonite, e) granite lithospheres.

```
% display tables of results

table(RockTHvals,RockQvals,'RowNames', name2, 'VariableNames', {'ThermalGradients', 'Gr
```

```
ans = 5×2 table
                                                                    ...
```

	ThermalGradients		
1 Columbia Diabase (exp)	23.7842	17.3823	40.4303

	ThermalGradients		
2 Columbia Diabase (synth)	50.7747	38.1154	76.4339
3 Venera 14	51.2255	38.0663	77.3848
4 quartz monzonite	48.9874	36.8281	76.1467
5 granite	46.0080	33.5198	69.4962

Table C. Geotherms (K/km) and Growth factors for polyphase models with log strain rates of -15 for dominant wavelength (coumn 1), interquartile range (columns 2, 3) and 20 km wavelength (column 4).

```
table(RockOptvals,'RowNames',name2)
```

ans = 5×1 table

	RockOptvals		
1 Columbia Diabase (exp)	23.7842	-15.0000	9.6318
2 Columbia Diabase (synth)	46.2747	-15.7000	51.9746
3 Venera 14	47.0000	-15.6726	49.2544
4 quartz monzonite	44.9874	-15.6000	44.2964
5 granite	42.5080	-15.4000	27.8720

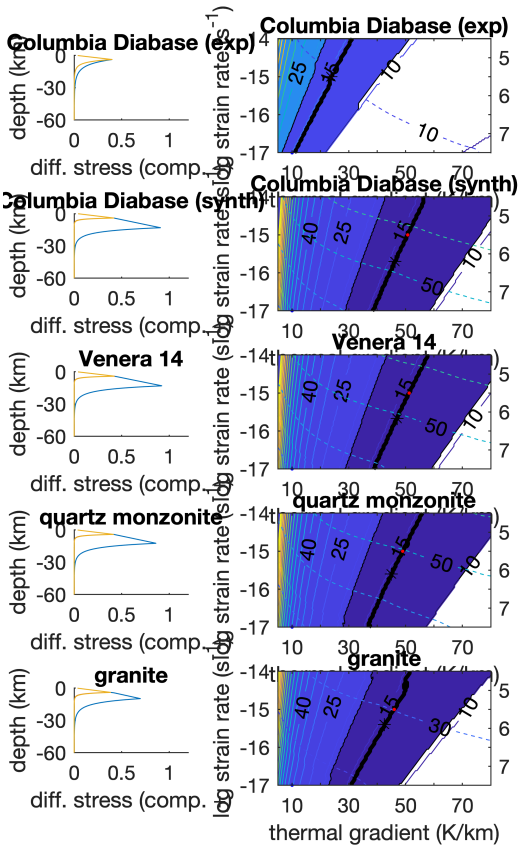
We use the modeled creep parameters to construct lithospheric strength profiles

```
% create christmas tree plot for representative values
% set up ranges of strain rate and thermal gradient to iterate over
% values based on B&G (1-2) and this study (3-7)
logexx2 = [-17, -15*ones(size(RockTHvals(:,1)')), RockOptvals(:,2)'];
thKkm2 = [10, RockTHvals(:,1)', RockOptvals(:,1)'];

% Create empty array to store numerical results
RockResults = zeros(length(Abar)*3, 6);

% loop over range of materials
for j = 1: length(Abar)
    subplot(7, 3, (3*j-2))
    hold on
    th = [thKkm2(1) thKkm2(j+1) thKkm2(j+length(Abar)+1)]./1000;
    Exx = 10.^[logexx2(1) logexx2(j+1) logexx2(j+length(Abar)+1)];
    for k = 1:length(th)
        [H, gam, eta, z, env] = bdt_plot(th(k), Ts, g, rhobar(j), Abar(j), nbar(j), Qba
        plot(-env/1000, -z/1000)
        dx = (z(2)-z(1));
        IntStrength = sum(dx.*(env(1:end-1) + env(2:end))./2) * 1e6; %N/m
        RockResults((j-1)*length(th)+k,:) = [log10(Exx(k)), th(k)*1000, H/1000, env(z==
    end
    hold off
    xlabel('diff. stress (comp. +)')
```

```
    ylabel('depth (km)')
%     legend([num2str(log10(Exx(1))),', ',num2str(th(1)*1000)],...
%         [num2str(log10(Exx(2))),', ',num2str(th(2)*1000)],...
%         [num2str(log10(Exx(3))),', ',num2str(th(3)*1000)],...
%         'Location', 'southeast')
    title(name2(j));
    axis([0 1.2 -60 0])
    yticks([-60, -30, 0])
    xticks([0, 0.5, 1])
end
```




```
% pos = fig2.Position;
% fig2.Position = [pos(1:3), 400];
```

```
table(RockResults, 'RowNames',...
    [name2(1) 'a' '1' name2(2) 'b' '2' name2(3) 'c' '3' name2(4) 'd' '4' name2(5) 'e' '
```

```
ans = 15×1 table
                                                                                ...
```

	RockResults			
1 Columbia Diabase (exp)	-17.0000	10.0000	3.8384	-395.8763
2 a	-15.0000	23.7842	3.7384	-398.9906
3 1	-15.0000	23.7842	3.7384	-398.9906
4 Columbia Diabase (synth)	-17.0000	10.0000	13.1802	-881.8197
5 b	-15.0000	50.7747	4.0802	-413.2378
6 2	-15.7000	46.2747	4.0802	-417.3010
7 Venera 14	-17.0000	10.0000	13.0704	-912.9689
8 c	-15.0000	51.2255	4.0704	-424.3183
9 3	-15.6726	47.0000	4.0704	-420.1209
10 quartz monzonite	-17.0000	10.0000	12.4228	-850.0224
11 d	-15.0000	48.9874	4.1228	-409.7476
12 4	-15.6000	44.9874	4.1228	-413.0569
13 granite	-17.0000	10.0000	9.8123	-680.6536
14 e	-15.0000	46.0080	4.0123	-353.8022
15 5	-15.4000	42.5080	4.0123	-364.3652

Table D. log strain rate (1/sec), geotherm (K/km), BDT (km), Stress at

BDT (MPa), viscosity at BDT (PaSec) and e-folding depth for various polyphase models

```
% pos = fig2.Position;
% axesj.Units = 'centimeters';
% axesj.OuterPosition = [0, -(j-1)*2.875, 3, 2.75];
```

Plot viscosity vs depth for power law vs exponential approximation

```
figS2 = figure(5);

% loop over range of materials
for j = 1: length(Abar)
    subplot(3, 2, (j))
    hold on
    z = RockResults(j*3-1,3)*1000:100:10000;
    T = Ts + z .* RockResults(j*3-1,2)./1000;
    visc = 1e6 .* 1/2*Abar(j).^(-1/nbar(j))...
        .* abs(10.^(RockResults(j*3-1,1))).^-(1-1/nbar(j))...
```

```
        .* exp(Qbar(j)./(nbar(j).*R.*T));
%     eta=1e6*1/2*Astar^(-1/n)*abs(Exx)^-(1-1/n)*exp(Q/(n*R*(Ts+th*H)))
    plot((visc),-z/1000,'r-')
    viscHat = RockResults(j*3-1,5).*exp(-RockResults(j*3-1,6).*(z-RockResults(j*3-1,3)).
    hold on
    plot((viscHat),-z/1000,'r:')
end
```

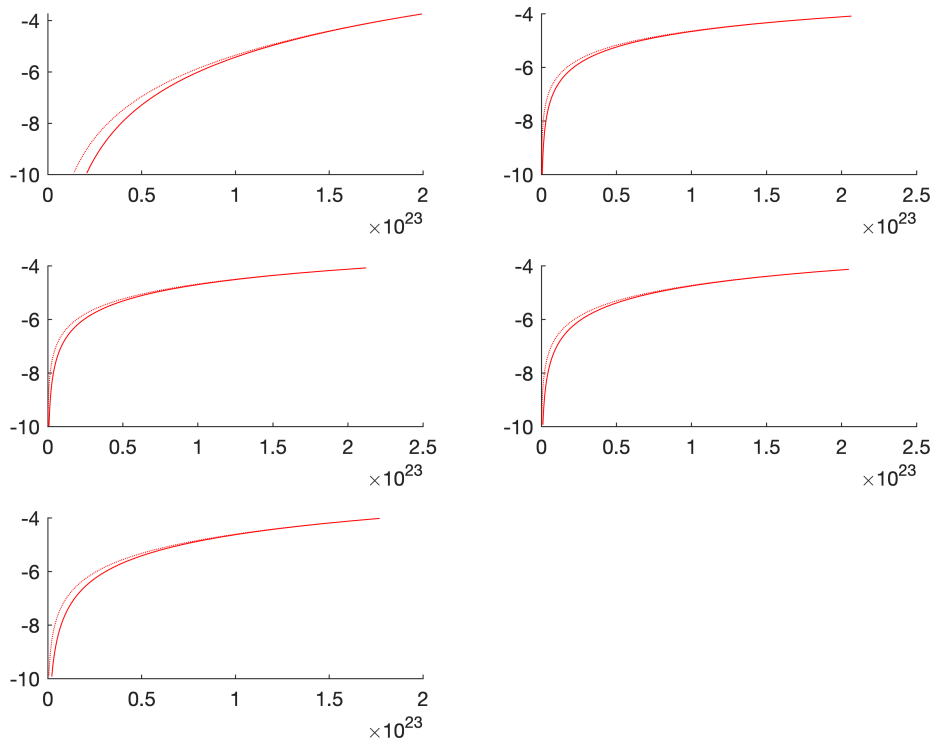


Figure S2. Viscosity decay with depth for polyphase models. Solid line is based on flow law (eq. 3) and dashed line is the approximation used in the model (eq. 4?)

Internal functions

Strength Envleope

```
function [H, gam, eta, z, env] = bdt_plot(th, Ts, g, rho, Astar, n, Q, Exx, phi)
% define brittle-ductile transition for given density (rho), gravity (g)
% thermal gradient (th), surface temperature (Ts),
% power-law rheology (Astar, n, Q), strain rate (Exx),
% and tectonic regime (phi = 'n' for normal, 'r' for reverse, 's' for
% strike slip)

% output is thickness of brittle layer (H), inverse e-folding depth (gam),
% and viscosity at the bdt (eta)
```

```
% Christmas tree plot is based on scripts written by John Townend (2001)

% define basic friction parameters for rock based on Byerlee (1978)
mu1=0.85;  %coefficient of friction (0-200 MPa)
mu2=0.6;        %coefficient of friction (>200 MPa)
F1=(sqrt(1+mu1^2)+mu1)^2;%frictional failure parameter (0-114 MPa)
F2=(sqrt(1+mu2^2)+mu2)^2;%frictional failure parameter (>114 MPa)

R=8.31446; % J/K/mol (NIST 2014)

% calculate depth to bdt

% calculate brittle strength
Ps=9.2; % surface pressure in MPa

% calculate depth of change in friction coefficient equivalent to 200 MPa
zt=(1.14e8-Ps*1e6)/(rho*g);

% set up two depth ranges with break at 200 MPa
z1=(zt:-100:0);
z1=fliplr(z1);
z2=(zt:100:100000);

% Calculate stress based on frictional strength in MPa
% Result depends on tectonic regime (phi)

if phi=='n' %normal faulting
    brit_S1_S31=(rho*g*z1*1e-6+Ps)*(F1-1)/F1;
    brit_S1_S32=(rho*g*z2*1e-6+Ps)*(F2-1)/F2;
    brit_S1_S32=brit_S1_S32+(brit_S1_S31(end)-brit_S1_S32(1)); % shift segments to meet

elseif phi=='r' %reverse faulting
    brit_S1_S31=(rho*g*z1*1e-6+Ps)*(F1-1);
    brit_S1_S32=(rho*g*z2*1e-6+Ps)*(F2-1);
    brit_S1_S32=brit_S1_S32+(brit_S1_S31(end)-brit_S1_S32(1)); % shift segments to meet

elseif phi=='s' %strike-slip faulting
    brit_S1_S31=2*(rho*g*z1*1e-6+Ps)*(F1-1)/(1+F1);
    brit_S1_S32=2*(rho*g*z1*1e-6+Ps)*(F2-1)/(1+F2);
    brit_S1_S32=brit_S1_S32+(brit_S1_S31(end)-brit_S1_S32(1)); % shift segments to meet

% if no tectonic regime is specified return error
else
    errordlg('Sorry! - The stress regime must be normal (n), strike-slip (s), or revers
end

% append brittle strength envelope to create single vector with compression
% negative
brit_S1_S3=-[brit_S1_S31 brit_S1_S32(2:end)];
z=[z1 z2(2:end)];

% calculate ductile strength
duct_S1_S3=-(abs(Exx)/Astar)^(1/n)*exp(Q./(n*R*(Ts+th*z)));
```

```
% find depth where ductile strength is less negative than brittle strength
% (crossover point)
[I]=find(duct_S1_S3>=brit_S1_S3, 1);
% set BDT to this point
H=z(I);

% calcualte inverse e-folding depth (gam) from the BDT depth, temperature,
% and rock rheology parameters
gam=Q*th/(n*R*(Ts+th*H)^2);

% calculate the viscosity at the BDT
eta=1e6*1/2*Astar^(-1/n)*abs(Exx)^-(1-1/n)*exp(Q/(n*R*(Ts+th*H)));

% store the envelope as the maximum of the [negative] strength values
env=max(duct_S1_S3, brit_S1_S3);
end
```

Stability analysis

```
function [qmax, Lmax, H]=perturb2(th, Ts, g, rho, Astar, n, Q, Exx, phi)
% function [qmax, kmax]=perturb(H, theta, Ts, Astar, n, Q, Exx)
% Implementation of Fletcher and Hallet (1983) necking/buckling model
% after implementation of Dombard and McKinnon (2001)
% qmax is the maximum growth rate of the instability of wave number kmax.
% th is the linear thermal gradient, Ts is the surface temperature, g is
% the gravitational acceleration, rho is the density, Astar, n, and Q are power-law rhe
% material constants, Exx is the horizontal strain rate.

% h=1;
qmax=-10^10; % very small initial value
n1=1e4; % plastic layer viscosity (approximating infinite)

% call subroutine to determine brittle layer thickness (H), inverse
% e-folding depth (gam) and interface viscosity (eta)

% [H, gam, eta] = bdt(th, Ts, g, rho, Astar, n, Q, Exx);
[H, gam, eta, ~, ~] = bdt_plot(th, Ts, g, rho, Astar, n, Q, abs(Exx), phi);

% Calculate S parameter from F&H
% tauy = 2*eta*abs(Exx);
% S = rho*g*H./(2*tauy);

% create vector of k values and empty q values
ks=1.4:0.05:4;
q = zeros(size(ks));

% iterate over wavenumber, k
for j = 1 : length(ks)
    k = ks(j);

    % initiaqlize P matrix that holds equation coefficients
    P=zeros(6,6);
```

```
    % make wavenmuber dimensionless
    lam=k/H;

    % initial small amplitude of topography (scaled to wavelength)
    h = .01*2*pi/lam;

    % define common terms for substrate
    m=gam/lam;
    a=sqrt((m^2/4+2/n-1+sqrt(m^4/16+m^2*(2/n+1)/2+1))/2);
    rr=sqrt(m^2/4+(n-1)/n^2);
    al=a-m/2;
    be=rr/a;

    % define common terms for surface layer
    al1=sqrt(1/n1);
    be1=sqrt(1-1/n1);

    % terms for stress equations
    M1=(4/n-1)*al-m*(1+al^2-be^2)-al*(al^2-3*be^2);
    M2=be*((4/n-1)-2*al*m+be^2-3*al^2);

    C=cos(be1*k);
    S=sin(be1*k);

    % set up matrics

    % first two rows are shear and normal stresses at surface due to
    % topography
    P(1,1)=(C-be1/al1*S)*exp(al1*k);
    P(1,2)=(be1/al1*C+S)*exp(al1*k);
    P(1,3)=(C+be1/al1*S)*exp(-al1*k);
    P(1,4)=-(be1/al1*C-S)*exp(-al1*k);
    P(2,:)=[C*exp(al1*k) S*exp(al1*k) -C*exp(-al1*k) -S*exp(-al1*k) 0 0];

    % next four are continuity equations for displacements (horizontal,
    % veritcal) and stresses (shear, normal) across interface
    P(3,:)=[al1 be1 -al1 be1 -al -be];
    P(4,:)=[1 0 1 0 -1 0];
    P(5,:)=[-2/n1 -2*al1*be1 -2/n1 2*al1*be1 1+al^2-be^2 2*al*be];
    P(6,:)=[2*al1 0 -2*al1 0 -M1 -M2];

    % vector of boundary conditions
    % (Sxz and Szz at surface; u, w, Sxz, Szz at interface)
    % Perturbation at surface
    BC=[2*n1*Exx*h -rho*g*h*H/(2*eta*al1*k) 0 0 0 0]';

    % solve for unkown coefficients using MATLAB mldivide
    A=P\BC;

%     % calculate vertical and horizontal velocities on grid
%     [X,Z] = meshgrid(-pi:pi/4:pi, 0:H/4:H);
%     W=((A(1)*cos(be1*lam*Z)+A(2)*sin(be1*lam*Z)).*exp(al1*lam*Z)+...
%         (A(3)*cos(be1*k)+A(4)*sin(be1*k))*exp(-al1*k)).*cos(lam*X);
```

```
    % calculate vertical velocity at (0 or pi,H)
    W0=sign(Exx)*((A(1)*C+A(2)*S)*exp(al1*k)+(A(3)*C+A(4)*S)*exp(-al1*k));

    % calculate growth rate factor
    q(j)=W0/(abs(Exx)*h); % corrected for contraction

    % check for max and if true store values
    if q(j)>qmax
        qmax=q(j);
        kmax=k;
    end
end

% calculate wavelength for maximum growth rate
Lmax=2*pi*H/kmax;
end
```

Mixing model

```
function [nbar, Qbar, Abar] = MPGe(phi, n, Q, A)
% function [nbar, Qbar, Abar] = MPGe(phi, n, Q, A)
%
% function form of Minimize Power Geometric Mean of Huet et al., 2014
% inputs are vectors of volume fraction (phi) and creep parameters
% (n, Q, A) for a polyphase rock composed. Output variables are averaged
% creep parameters.

% determine number of phases
nphase = length(phi);

% initialize ID variable with number for each phase
ID = 1:nphase;

% initialize "a" parameter (cumulative product of stress exponents of all
% other phases +1)
a = zeros(size(phi));

% loop over phases and calculate a
for j=1:nphase
    a(j)=prod(n(j~=ID)+1);
end

% calculate creep parameters

% average power law exponent (n)
nbar=(sum(phi.*a.*n))./(sum(phi.*a));

Qbar=(sum(phi.*a.*Q))./(sum(phi.*a));

Abar=prod((A).^((phi.*a)./sum(phi.*a)))...
    .*(sum((phi.*n)./(n+1))).^-nbar...
    .*prod((n./(n+1)).^((phi.*a.*n)./sum(phi.*a)));
```

```
end
```